Darius

$We\ choose\ \ (as\ in\ the\ draw)\ A=-i\ ,B=b\ ,C=-c\ ;\ The\ pentagon\ ABDEF\ is\ negatively\ oriented\ ,$

$ACGHI\ is\ positively\ oriented\ and\ BCJKL\ is\ positively\ oriented\ .We\ consider\ w=\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}\,.$

$Good\ to\ know:\ w^5=1\ and\ w^4+w^3+w^2+w+1=0\ .Let\ O\ be\ the\ center\ of\ ABDEF\ .We\ get\ that$

$$\frac{A-O}{B-O}=w\Rightarrow\frac{A-O}{B-A}=\frac{w}{1-w}\Rightarrow\frac{-i-O}{b+i}=\frac{w}{1-w}\Rightarrow O=-i-\frac{w(b+i)}{1-w}=\frac{-bw-i}{1-w}\ ;\ \frac{F-O}{A-O}=w\Rightarrow$$

$$\frac{F+\frac{bw+i}{1-w}}{-i+\frac{bw+i}{1-w}}=w\Rightarrow\frac{F+\frac{bw+i}{1-w}}{\frac{bw+iw}{1-w}}=w\Rightarrow F=\frac{b(w^2-w)+i(w^2-1)}{1-w}=-bw-i(w+1)\ .Due\ to\ the$$

$orientation\ of\ ACGHI\ we\ replace\ b\ with\ -c\ and\ w\ with\ \overline{w}=w^4\ ,get\ I=cw^4-i(w^4+1)\ .Let\ P\ be$

$$the\ center\ of\ BCJKL\ ;then\ \frac{C-P}{B-P}=w\Rightarrow\frac{-c-P}{b+c}=\frac{w}{1-w}\Rightarrow P=\frac{-c-bw}{1-w}\ \ .Now\ \frac{L-P}{B-P}=w^4\Rightarrow$$

$$\frac{L+\frac{c+bw}{1-w}}{b+\frac{c+bw}{1-w}}=w^4\Rightarrow L=\frac{b(w^4-w)+c(w^4-1)}{1-w}=b(-w^3-w^2-w)+c(-w^3-w^2-w-1)=$$

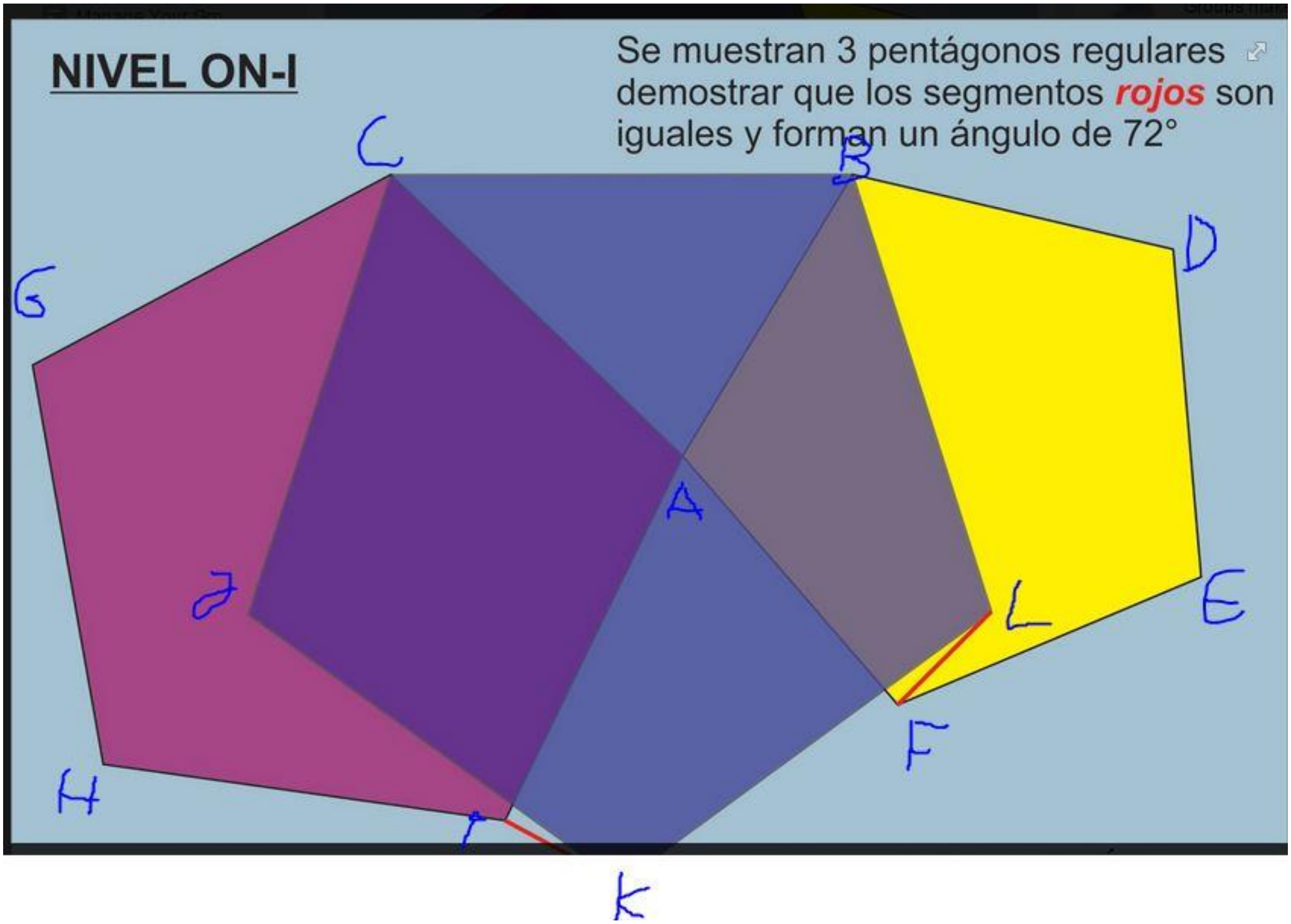
$$=b(w^4+1)+cw^4\ ;\ \frac{K-P}{C-P}=w^2\Rightarrow\frac{K+\frac{c+bw}{1-w}}{-c+\frac{c+bw}{1-w}}=w^2\Rightarrow K=\frac{b(w^3-w)+c(w^3-1)}{1-w}=$$

$=b(w^4+w^3+1)+c(w^4+w^3)\ .We\ get\ \overrightarrow{IK}=K-I=b(w^4+w^3+1)+cw^3+i(w^4+1)\ and\ \overrightarrow{FL}=$

$L-F=b(w^4+w+1)+cw^4+i(w+1)\ and\ we\ observe\ that\ w\cdot\overrightarrow{IK}=\overrightarrow{FL}\ wich\ gives\ us\ that\ IK=FL$

$and\ the\ two\ lines\ form\ a\ 72^{\circ}\ angle\ .$

Leo


NIVEL ON-I
Se muestran 3 pentágonos regulares
demostrar que los segmentos *rojos* son
iguales y forman un ángulo de 72°
C
B
D
G
A
J
L
E
F
H
I
K