Pinocchio

$We\ choose\ \ A(0\,,\sqrt{3})\,, B(-1\,,0)\,, C(1\,,0) \Longrightarrow K(3\,,0)\,;\ since\ O = G \Longrightarrow O\left(0\,,\frac{1}{\sqrt{3}}\right).But\ R = \frac{2}{\sqrt{3}} \Longrightarrow$

$(O):\ x^2 + \left(y - \frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3}\,;\ KO\ has\ the\ slope\ -\frac{1}{3\sqrt{3}} \Longrightarrow MN\ has\ the\ slope\ 3\sqrt{3}\ and\ since\ O \in MN\,,$

$then\ MN: y - \frac{1}{\sqrt{3}} = 3\sqrt{3}x \Longrightarrow y = \frac{1}{\sqrt{3}} + 3\sqrt{3}x\,.We\ make\ (O) \cap MN \Longrightarrow x^2 + 27x^2 = \frac{4}{3} \Longrightarrow x = \pm\frac{1}{\sqrt{21}} \Longrightarrow$

$M\left(\frac{1}{\sqrt{21}}\,,\frac{1}{\sqrt{3}} + \frac{3}{\sqrt{7}}\right)\ and\ N\left(-\frac{1}{\sqrt{21}}\,,\frac{1}{\sqrt{3}} - \frac{3}{\sqrt{7}}\right).Also\,, it's\ easy\ to\ see\ that\ \ D\left(0\,,-\frac{1}{\sqrt{3}}\right)\ and\ E\left(1\,,\frac{2}{\sqrt{3}}\right).$

$X \in (BE) \Longrightarrow X = (1-\alpha)B + \alpha E\,, \alpha \in (0\,,1) \Longrightarrow X\left(2\alpha - 1\,,\frac{2\alpha}{\sqrt{3}}\right)\,;\ KX^2 = KM^2 = \left(3 - \frac{1}{\sqrt{21}}\right)^2 +$

$+\left(\frac{1}{\sqrt{3}} + \frac{3}{\sqrt{7}}\right)^2 = \frac{32}{3} \Longrightarrow (\alpha - 2)^2 + \frac{\alpha^2}{3} = \frac{8}{3}\,;\ \alpha \in (0\,,1) \Longrightarrow \alpha = \frac{3-\sqrt{5}}{2} = \frac{1}{\varphi^2} \Longrightarrow 1 - \alpha = \frac{1}{\varphi}\ and\ since$

$X = (1-\alpha)B + \alpha E\,, then\ \frac{EX}{XB} = \varphi\,.$

$Now\ we\ consider\ the\ point\ Z' \in (AB)\,, Z' = (1-k)A + kB\,, where\ k = \frac{1}{\varphi^2} \Longrightarrow Z'\left(-\frac{1}{\varphi^2}\,,\frac{\sqrt{3}}{\varphi}\right).Obviousl$

$\frac{BZ'}{Z'A} = \varphi\,;\ we've\ got\ above\ that\ \ X\left(\frac{1}{\varphi^2} - \frac{1}{\varphi}\,,\frac{2}{\varphi^2\sqrt{3}}\right).For\ proving\ that\ Z = Z'\ it\ is\ eunough\ to\ show$

$that\ D\,, X\,, Z'\ are\ collinear\ \ i.e.\ \begin{vmatrix} -\frac{1}{\varphi^2} & \frac{\sqrt{3}}{\varphi} & 1 \\ \frac{1}{\varphi^2} - \frac{1}{\varphi} & \frac{2}{\varphi^2\sqrt{3}} & 1 \\ 0 & -\frac{1}{\sqrt{3}} & 1 \end{vmatrix} = 0 \leftrightarrow \varphi^3 = 2\varphi + 1\,, wich\ is\ true\,.$

$For\ second\ and\ 4-th\ part\ we'll\ consider\ Y'\ the\ midpoint\ of\ [XZ]\ and\ doing\ the\ same\ procedure\ as$

$above\ we\ get\ the\ desired\ conclusion\,.$

Leo