Lemma: For any function of two variables, say $f(x,y)$, we have

$$\max_y \min_x f(x,y) \le \min_x \max_y f(x,y), \tag{1}$$

with the equality, if and only if there exist x_0 and y_0 such that $\min_x f(x,y_0) = f(x_0,y_0) = \max_y f(x_0,y)$.

Proof: Note that for any x_0 and y_0 we have

$$\min_x f(x,y_0) \le f(x_0,y_0) \le \max_y f(x_0,y). \tag{2}$$

Minimizing over x_0, we get $\min_x f(x,y_0) \le \min_{x_0} \max_y f(x_0,y)$, and then maximizing over y_0, we have $\max_{y_0} \min_x f(x,y_0) \le \min_{x_0} \max_y f(x_0,y)$, which is equivalent to (1). If we have two equalities in (2), then we will also obtain an equality in (1) using the above arguments. Let's now assume the equality in (1), and let x_0, be such that $\min_x \max_y f(x,y) = \max_y f(x_0,y)$, and y_0, be such that $\max_y \min_x f(x,y) = \min_x f(x,y_0)$. Clearly,

$$\max_y \min_x f(x,y) = \min_x f(x,y_0) \le f(x_0,y_0) \le \max_y f(x_0,y) = \min_x \max_y f(x,y) \tag{3}$$

and from the assumed equality in (1), all terms in (3) are equal, which proves our necessary condition.