THE
GENTLEMAN'S
Mathematical Companion
FOR THE YEAR 1803:

CONTAINING

ANSWERS

TO

THE LAST YEAR'S ENIGMAS, REBUSSES, CHARADES,
QUERIES, AND QUESTIONS;

ALSO

New Enigmas,

Rebuſſes, Charades, Queries, and Queſtions,

PROPOSED TO BE

ANSWERED NEXT YEAR.

LIKEWISE

Two very curious and intereſting Papers, from the Tranſactions of the Royal Society of London.

Paper I.---" Reflections on the Communication of Motion by Impact and Gravity."

Paper II.---" Obſervations on the Limits of algebraical Equations; and a general Demonſtration of Deſcartes's Rule for finding their Number of affirmative and negative Roots."

THE ABOVE PAPERS ARE BY

THE REVEREND ISAAC MILNER,

(Now Dr. Milner) M.A., Fellow of Queen's College, Cambridge.

LONDON:
Printed by Knight and Compton, Middle-street, Cloth Fair,
FOR W. DAVIS, THE EDITOR.
Sold by H. D. Symonds, and W. Baynes, Paternoſter Row; and by Lackington, Allen, & Co., Temple of the Muſes.

1802

(*Price One Shilling and Sixpence.*)

required to determine their pofition when their diftance is equal to a given line?

16th Queftion (117), *by Mr. William Walker.*

Suppofe the fruftum of a cone whofe diameters and altitude are 50, 36, and 60, be cut by a plane paffing from the extremity of the greater bafe through the centre of gravity of the fruftum: it is required to find the area of the fection, and the folid content of the ungula contiguous to the greater end?

17th Queftion (118), *by M. F., of Reeth, Yorkfhire.*

In a certain Treatife on Fluxions it is afferted, that the fluent of

$$x^{mx}\dot{x} \text{ is} = x\left(1 - \frac{mx}{2^2} + \frac{m^2x^2}{3^3} - \frac{m^3x^3}{4^4} + \&c.\right)$$
$$+ mx^2\mathrm{L}x\left(\tfrac{1}{2} - \frac{mx}{3^2} + \frac{m^2x^2}{4^3} - \&c.\right)$$
$$+ \frac{m^2x^3\mathrm{L}^2x}{2}\left(\tfrac{1}{3} - \frac{mx}{4^2} + \frac{m^2x^2}{5^3} - \&c.\right)$$
$$+ \quad \&c. \quad (--- \&c. \quad - \quad - -)$$

Required the inveftigation?

18th Queftion (119), *by Mr. John Lowry.*

Given the vertical angle, the perimeter, and the fum of the bafe and perpendicular, to conftruct the triangle.

19th Queftion (120), *by Mr. John Whitley.*

To determine a point within a given polygon, fuch that, if perpendiculars be drawn therefrom to the fides, the fum of all the fquares formed upon them fhall be the leaft poffible.

20th Queftion (121), *by Mr. William Wallace, Perth.*

If from any two points B, E, in the circumference of a circle given in magnitude and pofition two right lines BCA, EDA, be drawn cutting the circle in C and D, and meeting in A; and from the point of interfection A to the centre of the circle AO be drawn, and the points E, C; B, D joined, and produced to meet an indefinite perpendicular erected at A on AO; then will FA be always equal to AG. Required the demonftration?

21ft Queftion (122), *by Mr. John Lowry.*

Given the vertical angle, the fum of the fides, and the fum or difference of the fquares of the bafe and perpendicular, to conftruct the triangle.

22d Queftion (123), *by Mr. George Sanderfon.*

Given the fum of the fides, the fum of the bafe and perpendicular, to conftruct the triangle, when the vertical angle is a maximum.

THE

GENTLEMAN'S Mathematical Companion

FOR THE YEAR 1804:

CONTAINING

ANSWERS

TO

The laſt Year's Enigmas, Rebuſſes, Charades, Queries, and Queſtions;

2

ALSO

NEW ENIGMAS, REBUSSES, CHARADES, QUERIES, AND QUESTIONS,

PROPOSED TO BE ANSWERED NEXT YEAR.

LIKEWISE

Two intereſting Papers,

From the Tranſactions of the Royal Society of London.

Paper I....“ An Inveſtigation of the Principles of Progreſſive and Rotatory Motion, by the Rev. S. Vince, A.M.”

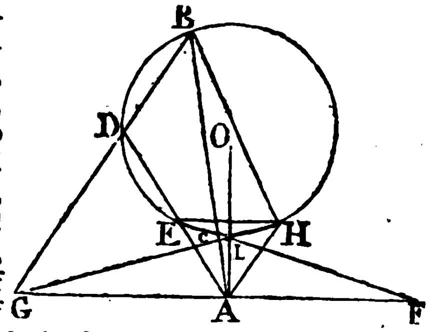
Paper II....“ A new Method of finding the Equal Roots of an Equation by Diviſion, by the Rev. John Hellins.”

The Diagrams engraved by Berryman.

LONDON:
Printed by Knight and Compton, Middle Street, Cloth Fair,
FOR W. DAVIS, THE EDITOR.
Sold by H. D. Symonds, and W. Baynes, Paternoſter Row; Lackington, Allen, and Co., Temple of the Muſes, Finſbury-ſquare; and by the Editor, 2, Albion Buildings, Alderſgate-ſtreet.

1803

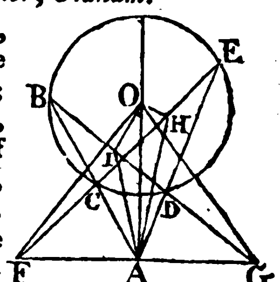
20th Queſtion (121) anſwered by Mr. W. Hilton.



Suppoſe the figure drawn as required, I being the point where AO interſects the circle; ſet off IH = IE, join G, H; A, H; B, H; and E, H, the laſt of which will be parallel to GF: now we have AH = AE, and the angle GAH = FAE. Moreover, becauſe EH is parallel to GF; and conſequently the angle EHA = AEH, we have HAG or the ſupplement of EHA = DEH, or the ſupplement of HBG; hence a circle will paſs through the four points A, H, B, and G, and the angle GHA = GBA = ſupplement of DEF = AEF; therefore the triangles FAE, GAH, are equal in all reſpects; or AF = AG. Q.E.D.

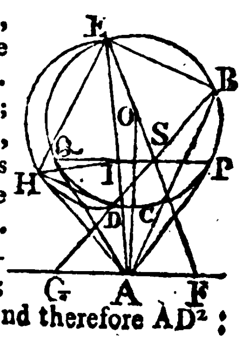
Scholium:----If AB fall on the other ſide of AO, the demonſtration may be effected in the ſame manner; likewiſe if the lines be drawn diagonally as through D, E; B, C, inſtead of through the adjoining points.

The ſame anſwered by Mr. John Fletcher, Oldham.



Let the lines be drawn as per theorem; join F, O, and G, O, and from the centre O let fall the ⊥s OH, OI, on the chords CE, BD, reſpectively; join A, H, and A, I; now the triangles AEC, ABD, are ſimilar, but H and I are the middle of their baſes; therefore the triangles AEH, ABI, will be alſo ſimilar, and the ∠ AID = AHC. Again; ſince OAG, OIG, are right angles, the points O, I, A, G, are in a circle, and for a ſimilar reaſon are the points O, H, A, F; therefore the ∠ AOG = AIG = AHF = AOF, and conſequently FA = AG. Q.E.D.

The ſame anſwered by Mr. John Whitley.

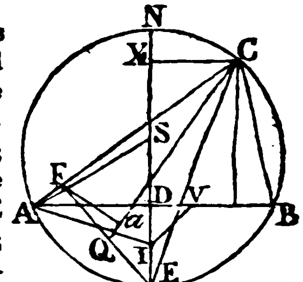


Demonſtr.:----Let the lines joining the points G, D, B; and C, E interſect in S; and from A draw the tangents AQ, AP; alſo draw P, Q, meeting DE in I. Upon DE as a diameter deſcribe the ſemicircle DHE; and draw IH perp. to DE, meeting it in H; join DH, EH, and AH; then becauſe the triangle AQP is iſoſceles, we have $AQ^2 = AI^2 + PIQ$; but by the circle, $PIQ = DIE = IH^2$; therefore AQ^2, or $AD \cdot AE = AI^2 + IH^2 = AH^2$ (Euc. 47, 1); conſequently AH is a tangent to the ſemicircle at H; wherefore the triangles ADH, AHE, are ſimilar; and therefore AD^2 :

$AH^2 = (AD . AE) :: DH^2 : EH^2 :: AD : AE$; but $DH^2 = ED . DI : EH^2 = ED . EI :: DI : IE$; wherefore $AD : AE :: DI : IE$. Again; it is well known that PQ will pafs through the point S, and be perpendicular to AO; and becaufe FG is alfo perp. to AO, therefore FG is parallel to PQ; wherefore the triangles EIS, EAF, are fimilar; and therefore $AE : AF :: EI : SI$; wherefore $AD : AF :: DI : SI$; but, by fimilar triangles, $DI : SI :: AD : AG$; wherefore $AD : AF :: AD : AG$; therefore $AF = AG$. Q.E.D.

Ingenious anfwers were alfo given by Meffrs. Cavill and Weftern.

21*ft Queftion* (122) *anfwered by Mifs Sophia Weftern.*



Analyfis:----1ft. When the fum of the fquares is given. Let ABC be the required triangle, and NDE the diameter of the circumfcribing circle drawn perpendicular to the bafe; draw CX parallel to AB, and bifect DX in S, and DE in I; then, if EF be drawn perpend. to AC, we have $FC^2 = EX . DN$; but the ratio of DN to DE is given, becaufe the vertical angle is given; therefore the rectangle $ED . EX$ is given (becaufe FC, half the fum of the fides, is given), or its fourth the rectangle $DI . IS$ is given. Join A, S; A, I; then $AD^2 + DS^2 = AS^2 =$ a fourth part of the given fum of the fquares of the bafe and perpendicular; therefore AS is given; and becaufe the ratio of AD to DE is given, the ratio of DA to DI will be given; and confequently the $\angle AID$ is given; alfo the ratio of AI to DI will be given; therefore the rectangle $AI . IS$ will be given; wherefore in the triangle AIS, there is given the bafe, the vertical angle, and the rectangle of the fides, to conftruct it; the method of doing which is well known.

2d. When the difference of the fquares is given, bifect FE in Q, join Q, C, and draw Fa perpendicular to it; then, taking $a^2 =$ the rectangle $SI . DI$, and $b^2 = SD^2 \backsim AD^2$, we have $SI : a :: a : DI$, or $SI^2 : a^2 :: a^2 : DI^2$, but $SI = SD + DI$, and $SD^2 = AD^2 \pm b^2$; therefore $SI^2 = SD^2 + DI^2 + 2SD . DI = AD^2 + DI^2 \pm b^2 + 2a^2$, and $AD^2 + DI^2 \pm b^2 + 2a^2 : a^2 :: a^2 : DI^2$. Now $AD^2 : DI^2 :: FC^2 : FQ^2 :: Ca : aQ$; therefore, by compofition, $AD^2 + DI^2 : DI^2 :: QC : aQ$; make $2a^2 \pm b^2 : c^2 :: QC : aQ$; then, by compofition, $AD^2 + DI^2 \pm b^2 + 2a^2 : DI^2 + c^2 :: QC : aQ :: a^2 : d^2$; then $DI^2 + c^2 : d^2 :: a^2 : DI^2$. Take $DV = c$; then $IV^2 : d^2 :: a^2 : DI^2$, or $IV : d :: a : DI$; therefore the rectangle of $IV . ID$ and the difference of their fquares are given to determine them. Hence the conftruction becomes eafy.

It was alfo anfwered by Meffrs. Cavill and Whitley.