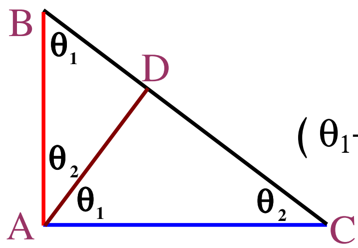
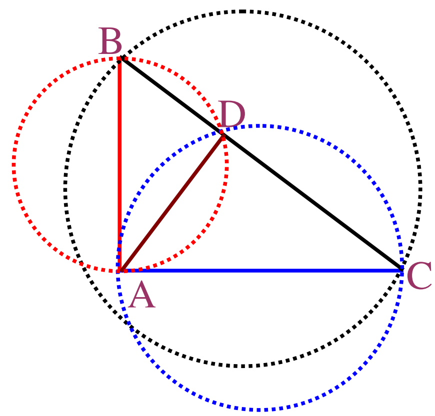
EL-MAKATY PROOF

We start with the original triangle ABC, A is right-angle.
And need only one additional construct – the altitude AD.
We drew a circle on (AB) diameter and a circle on (BC) diameter a circle on (AC) diameter.
We want proof that: $(BC)^2 = (AB)^2 + (AC)^2$
The triangles ABC, ABD, ADC are similar which leads to ratios:



$(\theta_1 + \theta_2 = 90^o)$



$$\frac{\Delta \text{Area}(ABC)}{\text{circle Area } (BC)} = \frac{\frac{1}{2} AC \times AB}{\pi(\frac{BC}{2})^2} = \frac{2AC \times AB}{\pi(BC)^2} \rightarrow 1$$

$$\frac{\Delta \text{Area}(ABD)}{\text{circle Area}(AB)} = \frac{\frac{1}{2} BD \times AD}{\pi(\frac{AB}{2})^2} = \frac{2BD \times AD}{\pi(AB)^2} \rightarrow 2$$

$$\frac{\Delta \text{Area}(ADC)}{\text{circle Area}(AC)} = \frac{\frac{1}{2} CD \times AD}{\pi(\frac{AC}{2})^2} = \frac{2CD \times AD}{\pi(AC)^2} \rightarrow 3$$

From 1, 2 $$\frac{2AC \times AB}{\pi(BC)^2} = \frac{2BD \times AD}{\pi(AB)^2}$$

$$AC\times AB\times AB^2 = BD\times AD\times BC^2 \quad \rightarrow 4$$

From 1, 3 $\quad \dfrac{2AC\times AB}{\pi(BC)^2} = \quad = \dfrac{2CD\times AD}{\pi(AC)^2}$

$$AC\times AB\times AC^2 = CD\times AD\times BC^2 \quad \rightarrow 5$$

sum up $(4+5)$

$$AC\times AB\times AB^2 + AC\times AB\times AC^2 = BD\times AD\times BC^2 + CD\times AD\times BC^2$$

$$AC\times AB\times (AB^2 + AC^2) = AD\times BC^2(BD+CD) = BC^2\times AD\times BC$$

$$(AB^2 + AC^2) = \frac{BC^2\times AD\times BC}{AC\times AB} \quad \rightarrow 6$$

$$\frac{1}{2}AC\times AB = \frac{1}{2}BC\times AD = \Delta \text{ Area (ABC)}$$

$$AC\times AB = BC\times AD \quad \rightarrow 7$$

From (7) and (6) we get :

$$(AB^2 + AC^2) = \frac{BC^2\times AC\times AB}{AC\times AB}$$

$$(AB^2 + AC^2) = BC^2$$